PG – 715

**II Semester M.Sc. Degree Examination, June 2016**
**(R.N.S.) (2011 – 12 and Onwards)**
**MATHEMATICS**
**M – 204 : Partial Differential Equations**

Time : 3 Hours  Max. Marks : 80

***Instructions*** : *1) Answer **any five** questions choosing atleast **two** from **each** Part.*
*2) **All** questions carry **equal** marks.*

PART – A

1. a) Define linear, semi-linear, quasi linear and non-linear partial differential equation of first order. Explain the method of characteristics for semi-linear equation $a(x, y)\, u_x + b(x, y)\, u_y = c(x, y, u)$. **6**

   b) Solve the following by the method of characteristics :

   i) $x\, u_x + (x + y)\, u_y = u + 1$ with $u = x^2$ on $y = 0$

   ii) $y\, u_x + x\, u_y = u$ with the data $u(x, 0) = x^3$, $u(0, y) = y^3$. **6**

   c) Solve the Cauchy problem $(u^2 - y^2)\, u_x + x \cdot y\, u_y + xu = 0$ with $u = y = x$, $x > 0$. **4**



2. a) Solve the IVP $u_t + u_x\, u = 0$ with initial data $u(x, 0) = \begin{cases} 1 & \text{for} \quad x < 0 \\ 1 - x & \text{for} \quad 0 \le x \le 1 \end{cases}$. **5**

   b) Solve the non-linear Cauchy problem $u_t + u\, u_x = au$, $x \in R$, $t > 0$; $u(x, 0) = bx$ is $u = ab\, x\, e^{-at}$. **6**

   c) Solve the non-linear Cauchy problem $pq = u$ with $u = 1$ on $y = -x$. **5**

3. a) Classify and reduce the following equation into its canonical form given that $x^2 u_{xx} - 2xy\, u_{xy} + y^2 u_{yy} = \sin x$. **6**

   b) Classify the equation into hyperbolic, parabolic and elliptic equations $u_{xx} + x\, u_{yy} = 0$ for $x \neq 0$. Reduce it to canonical form when $x > 0$. **5**

   c) Obtain the general solution of $u_{xx} = u_{yy}$. **5**

**P.T.O.**

4. a) Solve the linear equations :

i) $(D - D' - 1)(D - D' - 2)\, z = e^{2x+y} \left( D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y} \right)$,

ii) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial z}{\partial y} = x^2 + y^2$. **8**

b) Solve the following equations by Monge's method

i) $2x^2 r - 5xy\, s + 2y^2 t + 2\,(px + qy) = 0$,

ii) $(x - y)\,(xr - (x + y)\, s + yt) = (x + y)\,(p - q)$. **8**

PART – B

5. a) Using an integral transform find the solution of the following :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2};\ 0 < x < \infty,\ t > 0,$$

$$\left.\begin{aligned} u(0, t) &= f(t) \\ \frac{\partial u}{\partial x}(\infty, t) &= 0 \end{aligned}\right\},\ t > 0,$$

$$\left.\begin{aligned} u(x, 0) &= 0 \\ \frac{\partial u}{\partial t}(x, 0) &= 0 \end{aligned}\right\},\ 0 < x < \infty.$$ **8**



b) Show that separation of variables method when used in the case of three-dimensional wave equation in cylindrical polar co-ordinates leads to the classical Bessel equation. **8**

6. a) State the Dirichlet problem, involving a Laplace equation for a circle, and solve the same. **8**

b) Solve by the method of separation of variables the BVP $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < a,$

$0 < y < b$

$$\left.\begin{aligned} u(x, 0) &= x \\ u(x, b) &= 0 \end{aligned}\right\},\ 0 < x < a,$$

$$\left.\begin{aligned} u(0, y) &= 0 \\ u(a, y) &= 0 \end{aligned}\right\},\ 0 < y < b.$$ **8**

7. a) Obtain the solution of the Cauchy problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; $-\infty < x < \infty$, $t > 0$,

$u(x, 0) = f(x)$, $-\infty < x < \infty$ by using infinite Fourier transform. **8**

b) Illustrate Green's function method of solving a certain class of PDEs. Use a suitable example from hyperbolic or elliptic or parabolic equations. **8**

8. a) Using any successive approximation method find a two-term solution of the following : $x\,u_x + u_y = u^2$; $u(x, 0) = x^2$, $u(0, y) = 0$. **8**

b) Make a weak formulation of the BVP $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -u^2 - 1$, $0 < x < a$, $0 < y < b$,

$$\left.\begin{aligned} u(0, y) &= 0 \\ u(a, y) &= 0 \end{aligned}\right\}, \; 0 < y < b,$$

$$\left.\begin{aligned} u(x, 0) &= 0 \\ u(x, a) &= 0 \end{aligned}\right\}, \; 0 < x < a$$

Outline the procedure of solving the weak formulation. **8**



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